Trading strategies based on ARIMA prediction and Markowitz's theory

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Keywords: ARIMA Prediction, Smoothing Index Prediction, Markowitz's theory, Trading strategy.

Abstract: This paper establishes a mathematical model to study the trading strategy. Firstly, we establish the Gold Price ARIMA Prediction Model Bitcoin Price Smoothing Index Prediction Model to predict the future price of gold and bitcoin, respectively. The price of any subsequent day can be predicted with relative precision just from the price data before that day. Secondly, we established a set of judgments of the best point of buying and selling selection. Based on the price prediction, we can judge whether a particular day is the best buying and selling point. Thirdly, we built Risk Asset Portfolio Selection Model on Markowitz's theory. The model can get an optimal proportion of assets to control risks and increase returns by considering the benefits and risks of a single asset and the relationships among various assets in the portfolio.

1. Introduction

We need to develop a model to give the best daily trading strategy based only on price data up to that day.

In our view, trading strategies include the points of buying and selling and the ratio of buying and selling. If we are traders, we know when to buy or sell by looking at future prices. Therefore, the best buy and sell point should be judged based on prediction. As for the buy-to-sell ratio, we should establish an investment matching model that considers the trader's psychology to solve this problem. Most investors hope to control risks and gain profits, so our investment matching model should be aimed at this point.

2. Model Establish and Solution

2.1 Gold Price ARIMA Prediction Model

We mainly consider trend, periodicity, and randomness factors that affect gold price changes.

$$Y_i^g = T_i^g + C_i^g + I_i^g \tag{1}$$

Y_i^g: total prediction of gold price in the *i*th period.

T_i^g: trend prediction of gold price in the *i*th period.

C_i^g: periodicity prediction of gold price in the *i*th period.

I_i^g: total prediction of gold price in the *i*th period.

We used the moving average method [3] to calculate the trend prediction of the gold price, and the period of movement is 30 days.

Primary moving average:

$$S_t' = \frac{R_t^g + R_{t-1}^g + R_{t-2}^g + \ldots + R_{t-30+1}^g}{30} \tag{2}$$

R_i^g: actual gold price in the *i*th period.

Second moving average:

$$S_t'' = \frac{S_t' + S_{t-1}' + S_{t-2}' + \ldots + S_{t-30+1}'}{30} \tag{3}$$

Predictive linearity coefficient:

$$a_t = 2S_t' - S_t'' \tag{4}$$

$$b_t = \frac{2}{30-1}(S_t' - S_t'') \tag{5}$$

Trend prediction of gold price:

$$T_{t+i}^g = a_t + b_t i \tag{6}$$

We calculated the predicted trend value and drew a line chart, as shown in Figure 1.

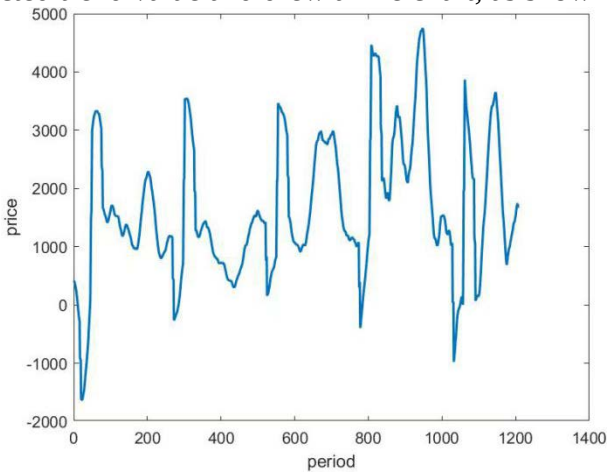

Figure 1 Predicted trend value

The predicted value function of the trend prediction is:

$$T_i^g = 1.876i + 1117 \tag{7}$$

We make the difference between the trend prediction of gold price and the actual value and draw the difference area chart in Figure 3. It can be concluded that the trend forecast value alone cannot accurately predict gold price changes.

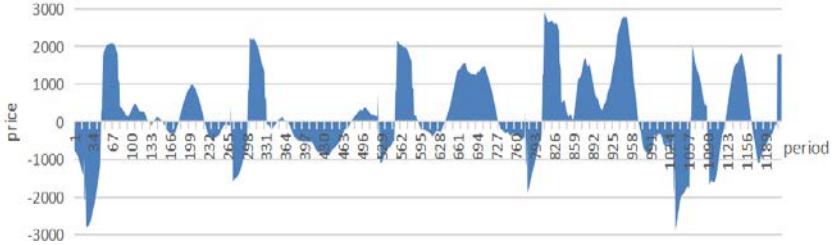

Figure 2 The trend prediction of gold price and the actual value

Due to periodic and random factors, the trend prediction of gold price can not be completely used as the forecast value of gold price. Based on the trend prediction value, we need to add periodic prediction values and random prediction values to make the prediction more accurate. The relative formula is as follows:

$$\Delta I_i^g = T_i^g - R_i^g = C_i^b + I_i^g \tag{8}$$

We conducted residual graph analysis and stationarity test and found that ARIMA (2,1,2) was the most suitable ARIMA model [4] for simulating gold price. Firstly, we established a model based on the five years' data, which took trend, periodicity, and randomness factors into account. We tested the fit degree, and R^2 was greater than 0.9. The predicted and actual values overlap, so the model fit degree is good.

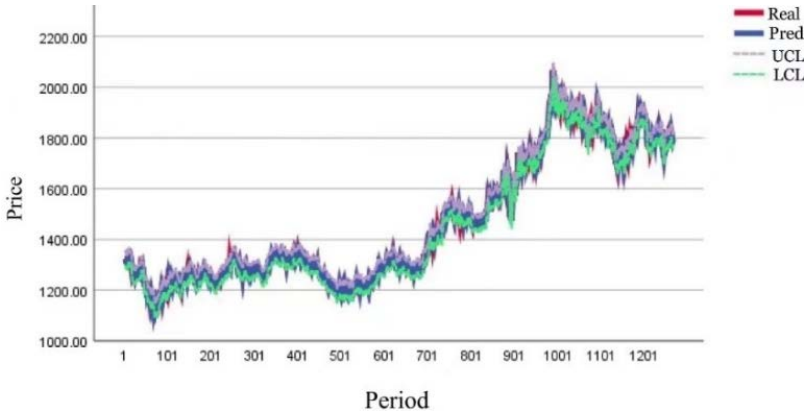

Figure 3 ARIMA prediction

The five-year prediction function of the gold price is as follows:

$$Y_i^g = 1.876i - 109.5cos(7.116 \times 10^{-3} i) - 88.99sin(7.116 \times 10^{-3} i) + 1153.54 + I_i^g \quad (9)$$

The model considers trend, periodicity, and randomness factors that affect the change of gold price. We need to analyze the three factors separately. Firstly, we analyzed the trend factors. According to Equation (9), gold price shows an upward trend in five years with a small fluctuation range, which is suitable for long-term investment.

Secondly, we analyze the periodicity factors. We fit the periodic predictive value function:

$$C_i^g = 36.54 - 109.5cos(7.116 \times 10^{-3} i) - 88.99sin(7.116 \times 10^{-3} i) \quad (10)$$

The periodic prediction chart is drawn as shown in Figure 4.

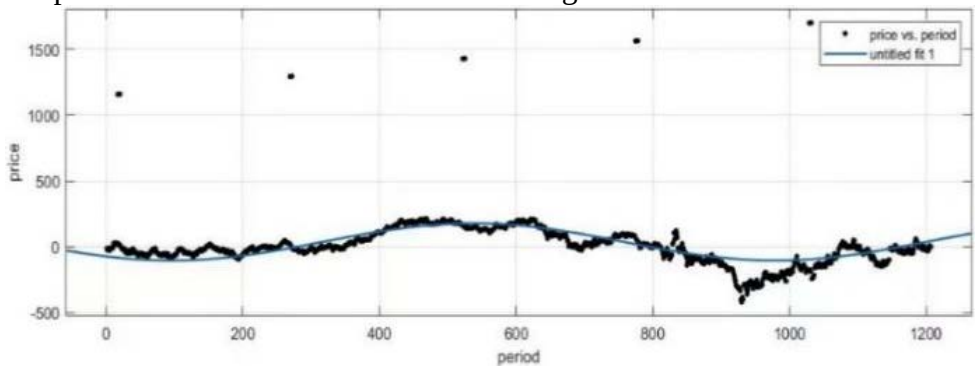

Figure 4 Periodic prediction

The blue curve in Figure 4 is the periodic prediction. As can be seen from the Figure 4, the gold price fluctuation experienced 1.5 cycles in five years. In the same periodicity, the price of gold shows a trend of rising-falling-rising. When investing, we can buy on the down part of a cycle and sell on the up.

We have a deeper understanding of gold price changes in the past five years through the above model analysis. Through the experiment of five years of gold price data, we believe that the ARIMA time series prediction model is suitable for gold price prediction. However, in real life, investors can not know the future price of gold and can only use the one-time price to predict the future price. We establish the dynamic ARIMA time series prediction model to enable investors to predict the future price every day.

With the dynamic ARIMA time series prediction model, we can only use the gold trading data up to that day to predict the gold movement after that day. This model can choose the date we predict and the number of predicted days after that. By looking ahead, traders can determine whether the day is a buy or sell. This model predicts the future 20 days trading situation on the 30th day, as shown in Figure 5. R^2 was greater than 0.9, so the model fit degree is good.

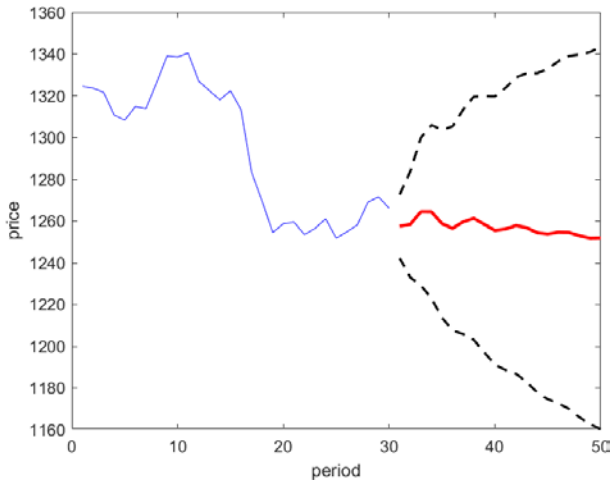

Figure 5 Dynamic ARIMA prediction

2.2 Bitcoin Price Smoothing Index Prediction Model

We also mainly consider trend, periodicity, and randomness factors that affect bitcoin price changes.

$$Y_i^b = T_i^b + C_i^b + I_i^b \tag{11}$$

Y_i^b: total prediction of bitcoin price in the *i*th period.

T_i^b: trend prediction of bitcoin price in the *i*th period.

C_i^b: periodicity prediction of bitcoin price in the *i*th period.

I_i^b: random prediction of bitcoin price in the *i*th period.

We established a smoothing index prediction model [4] for five years (as shown in Figure 8), which considered trend factors, periodicity factors, and randomness factors simultaneously. We tested the fit degree, and R^2 was greater than 0.9. The predicted and actual values overlap, so the model fit degree is good.

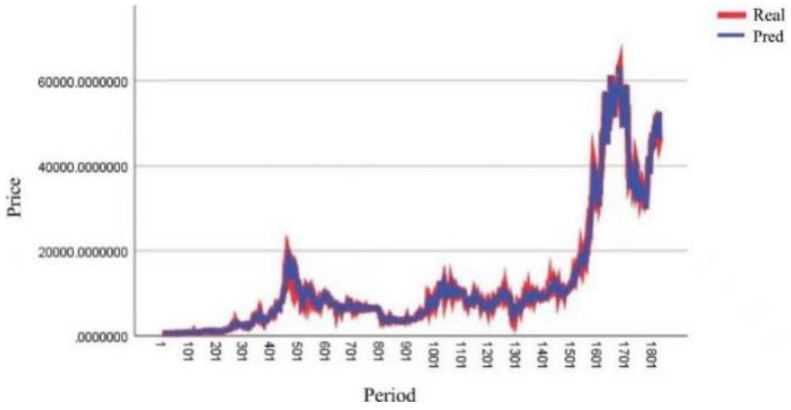

Figure 6 Smoothing index prediction

We get the five-year price forecast of bitcoin as follows:

$$Y_i^b = 486.1e^{2.566\times10^{-3}i} - 5223cos(1.095\times10^{-2}i) - 1061sin(1.095\times10^{-2}i) + 931.5 + I_i^b \tag{12}$$

The model considers trend, periodicity, and randomness factors that affect bitcoin price change. We need to analyze the three factors separately. Firstly, the trend factors are analyzed, and the calculation formula is as follows:

First exponential smoothing value:

$$S_t' = 0.3R_i^b + 0.7S_{t-1}' \tag{13}$$

R_i^g: actual gold price in the *i*th period.

The quadratic exponential smoothing value:

$$S_t^{II} = 0.3S_t^I + 0.7S_{t-1}^{II} \tag{14}$$

Predicted linear exponential coefficient value:

$$a_t = 2S_t^I - S_t^{II} \tag{15}$$

$$b_t = \frac{0.3}{1-0.3}(S_t^I - S_t^{II}) \tag{16}$$

Trend prediction:

$$T_{t+i}^b = a_t + b_t i \tag{17}$$

The bitcoin price trend term expression is a monotonically increasing exponential function. Therefore, it can be seen that the overall trend of bitcoin price in five years shows explosive growth and great volatility.

In addition to trend factors, there are also periodic and random factors affecting the price of bitcoin:

$$\Delta I_i^b = T_i^b - R_i^b = C_i^b + I_i^b \tag{18}$$

We conducted a suitable combination of periodic factors to draw a curve, as shown in Figure 7.

$$C_i^b = 931.5 - 5223cos(1.095 \times 10^{-2} i) - 1061sin(1.095 \times 10^{-2} i) \tag{19}$$

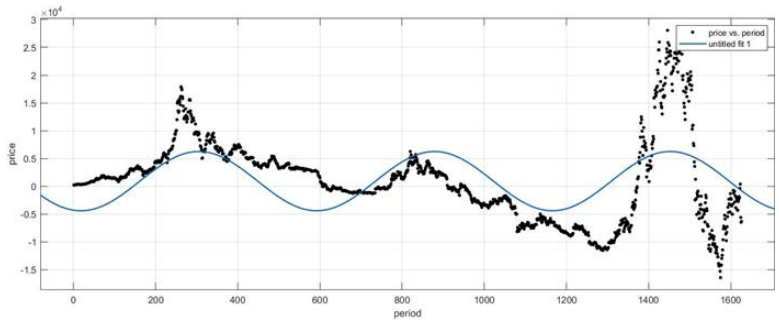

Figure 7 Fitting combination of periodic factors

The blue curve is the periodic predicted value. As can be seen from the figure, the price change of bitcoin in 5 years presents 3 cycles, and in the same cycle, bitcoin price presents a trend of rising-falling-rising. Bitcoin price changes are shorter and more volatile than gold, making it more suitable for short-term investments.

For investors, it can predict the future price of bitcoin every day, and we established a dynamic smooth exponential prediction model. Like the gold price dynamic ARIMA prediction model, the Bitcoin dynamic Smooth index prediction model can analyze the bitcoin price data before a specific day and then predict the price data of any subsequent day. This model predicts the trading situation for the next 40 days on the 1000th day, as shown in Figure 8. R^2 is greater than 0.9, the fitting effect is good.

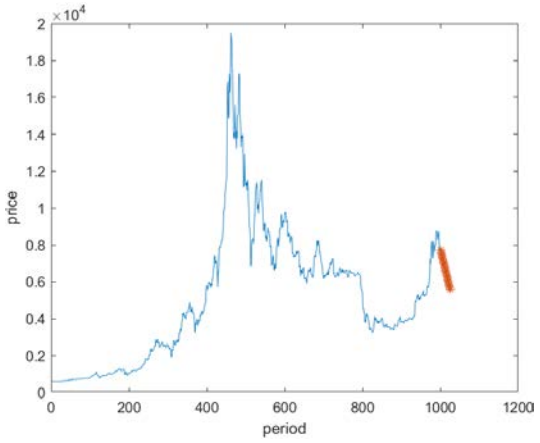

Figure 8 Fitting combination of periodic factors

2.3 Judgment of the best point of buying and selling

After completing the establishment of the Gold Price ARIMA Prediction Model and the Bitcoin Price Smoothing Index Prediction Model, we can predict asset prices after any day in five years and only use the price data before that day. Based on the prediction of the first two models, we establish a set of the best point of buying and selling selection mechanisms.

The predicted yield of gold in the *i*th period:

$$\alpha_i^{g'} = \frac{Y_i^g - Y_{i-1}^g}{Y_{i-1}^g} \tag{20}$$

The predicted yield of Bitcoin in the *i*th period:

$$\alpha_i^{b'} = \frac{Y_i^b - Y_{i-1}^b}{Y_{i-1}^b} \tag{21}$$

For bitcoin, if the predicted return on a given day is greater than 1%, we recommend traders to buy it. If the predicted return on a given day is less than -1.5%, and the value of bitcoin is greater than the purchase price (including 2% of the transaction cost), we recommend traders sell it.

We can get the daily forecast trend function based on the gold dynamic ARIMA prediction model. We can judge whether to buy or sell gold by the size of the trend growth coefficient. If the trend growth coefficient is greater than the critical value on any given day, traders are advised to buy, and if the trend growth coefficient is less than the critical value on any given day, traders are advised to sell.

2.4 Risk Asset Portfolio Selection Model

We chose to use Sharpe Ratio [1] to measure the risk and return of a risky asset comprehensively. We measure return by mean of return rate and risk by the variance of return rate.

Sharpe Ratio:

$$S_p = \frac{E(r_p) - r_f}{\sigma_p} \tag{22}$$

r_f: return on fixed income assets.

Actual yield of gold in the *i*th period:

$$\alpha_i^g = \frac{R_i^g - R_{i-1}^g}{R_{i-1}^g} \tag{23}$$

Actual yield of Bitcoin in the *i*th period:

$$\alpha_i^b = \frac{R_i^b - R_{i-1}^b}{R_{i-1}^b} \tag{24}$$

By maximizing the Sharpe ratio, the optimal asset purchase weight can be calculated [1]:

$$\omega_i^g = \frac{E(\alpha_i^g)\sigma^2(\alpha_i^b) - E(\alpha_i^g)\mathrm{Cov}(\alpha_i^g, \alpha_i^b)}{E(\alpha_i^g)\sigma^2(\alpha_i^b) + E(\alpha_i^b)\sigma^2(\alpha_i^g) - [E(\alpha_i^g) + E(\alpha_i^b)]Cov(\alpha_i^g, \alpha_i^b)} \tag{25}$$

$$\omega_i^b = 1 - \omega_i^g \tag{26}$$

We calculate the optimal weight, and the optimal asset purchase proportion is 62.5% for gold and 37.5% for bitcoin.

2.5 Test of superiority

To verify that our trading strategy has a good risk resistance. We calculated the yield correlation between gold and bitcoin. According to the Markowitz model [1], we draw the following curve:

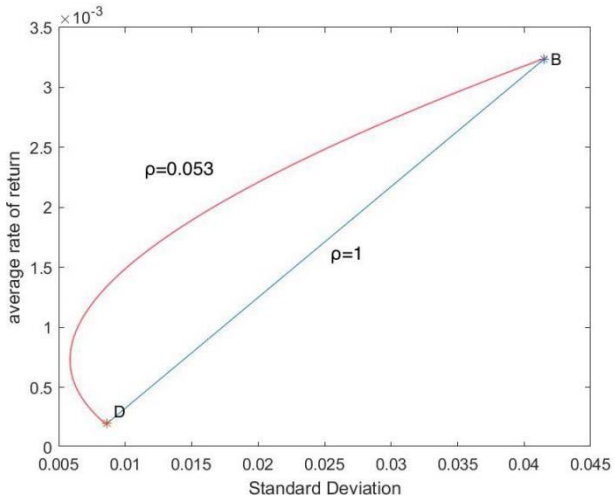

Figure 9 Markowitz model curve

According to Figure 9, we can make the following analysis: Gold slightly positively correlates with bitcoin yields. They have more than one point in the opportunity portfolio curve, lower risk than gold or Bitcoin, and higher yields than either. It shows that our model plays a good role in risk diversification.

3. Model Evaluation

3.1 Strengths

The model is highly usable. Users of the model can make predictions and judgments about prices beyond any given day for five years. The model is highly accurate. In the analysis of model 1 and Model 2, we used the model for prediction many times, and its fitting degree was greater than 0.9.
The model considers various factors affecting price changes, including trend factors, periodic factors, and random factors.

The model considers the psychological preference of most investors and reasonably quantifies the risk and return objectives.

3.2 Weaknesses

Due to the limitation of sample data, our model can only be used for 2017-2021. We can use more data modeling in the future to make the model more widely applicable.

The model completely separates bitcoin from gold accounts without consideration for interactive investments.

The model does not consider other factors except trend, periodicity, and randomness. We can add more influence factors to modify the model to make it more accurate.

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